A vector a mathematical abstraction allowing us to represent \( \mathbf{v} \) and \( \mathbf{w} \) in space.

A (2-dimensional) vector \( \mathbf{v} \) with components \( a \) and \( b \) is written
\[
\mathbf{v} = \begin{bmatrix} a \\ b \end{bmatrix} \quad \text{(row notation)} \quad \text{or} \quad \mathbf{v} = a\mathbf{i} + b\mathbf{j} \quad \text{(standard basis notation)}.
\]

The magnitude of \( \mathbf{v} \) is its length. It is given by
\[
\|\mathbf{v}\| = \sqrt{a^2 + b^2}.
\]

The vector from point \( P = (a_1, b_1) \) to \( Q = (a_2, b_2) \) is given by
\[
\mathbf{v} = \overrightarrow{PQ} = \begin{bmatrix} a_2 - a_1 \\ b_2 - b_1 \end{bmatrix}.
\]

The magnitude of this vector is the same as the distance from \( P \) to \( Q \).

Given vectors \( \mathbf{v} = (a_1, b_1) \) and \( \mathbf{w} = (a_2, b_2) \) we define the sum of \( \mathbf{v} \) and \( \mathbf{w} \) as
\[
\mathbf{v} + \mathbf{w} = \begin{bmatrix} a_1 + a_2 \\ b_1 + b_2 \end{bmatrix}
\]
and given a scalar \( \lambda \), then the scalar product of \( \mathbf{v} \) by \( \lambda \) is
\[
\lambda \mathbf{v} = \lambda a\mathbf{i} + \lambda b\mathbf{j}.
\]

We call \( \mathbf{v} \) and \( \mathbf{w} \) parallel if \( \mathbf{w} = \lambda \mathbf{v} \) for some scalar \( \lambda \neq 0 \). Note that parallel vectors point in the same, or opposite direction.

We can express a line \( L \) through a point \( P = (x_0, y_0) \) with direction vector \( \mathbf{v} = (a, b) \) by
\[
\mathbf{r}(t) = 
\begin{bmatrix} x_0 + at \\ y_0 + bt \end{bmatrix}.
\]
Where \( O \) is the origin \((0,0)\). This allows us to parametrize the line \( L \) by the parametric equations:
\[
x = x_0 + at, 
\quad y = y_0 + bt
\]

The axes in \( \mathbb{R}^3 \) are labeled so they satisfy the right-hand rule: When the fingers of your right hand curl from the positive \( x \)-axis towards the positive \( y \)-axis, your thumb points in the positive \( z \)-direction.
1. Find the components of $\vec{PQ}$ where $P = (1, -3)$ and $Q = (2, 7)$.

2. Let $\mathbf{v} = \langle 6, 9 \rangle$. Which of the following are parallel to $\mathbf{v}$ and which point in the same direction.

   (a) $\langle 12, 18 \rangle$
   (b) $\langle 3, 2 \rangle$
   (c) $\langle 2, 3 \rangle$
   (d) $\langle -6, -9 \rangle$
   (e) $\langle -24, -27 \rangle$
   (f) $\langle -24, -36 \rangle$.

3. What is the difference between $(a,b)$ and $\langle a,b \rangle$?

4. You are likely used to expressing a line as $y = mx + b$. What advantages does this have over the vector representation, what advantages does the vector representation have?

5. The concepts for 2-dimensional vectors generalize to 3-dimensional vectors by just adding a third component.

   (a) Express the 3-dimensional vector with components $a,b,c$ in both row notation, and as a sum of standard basis elements $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

   (b) Express the vector $\mathbf{v}$ from point $P = (a_1, b_1, c_1)$ to $Q = (a_2, b_2, c_2)$ as a row vector. What is the distance from $P$ to $Q$? What is the magnitude of $\mathbf{v}$?

   (c) What is the sum of two vectors $\mathbf{v} = \langle a_1, b_1, c_1 \rangle$ and $\mathbf{w} = \langle a_2, b_2, c_2 \rangle$? What is their difference? What is the scalar product of $\mathbf{v}$ by $\lambda$?

6. Find a vector parametrization for the line the passes through $P = (4, 0, 8)$ with direction vector $\mathbf{v} = (1, 0, 1)$.

7. Find parametric equations for the line perpendicular to the $xz$-plane, passing through the point $(1, -1, 2)$.

8. Which of the following ways of labeling axes follow the right hand rule?
• A vector a mathematical abstraction allowing us to represent magnitude and direction in space.

• A (2-dimensional) vector \( \mathbf{v} \) with components \( a \) and \( b \) is written

\[
\mathbf{v} = \begin{pmatrix} a \\ b \end{pmatrix} \text{ (row notation) or } \mathbf{v} = a\mathbf{i} + b\mathbf{j} \text{ (standard basis notation).}
\]

• The magnitude of \( \mathbf{v} \) is its length. It is given by

\[
\| \mathbf{v} \| = \sqrt{a^2 + b^2}
\]

• The vector from point \( P = (a_1, b_1) \) to \( Q = (a_2, b_2) \) is given by

\[
\mathbf{v} = \overrightarrow{PQ} = \begin{pmatrix} a_2 - a_1 \\ b_2 - b_1 \end{pmatrix}
\]

The magnitude of this vector is the same as the distance from \( P \) to \( Q \).

• Given vectors \( \mathbf{v} = (a_1, b_1) \) and \( \mathbf{w} = (a_2, b_2) \) we define the sum of \( \mathbf{v} \) and \( \mathbf{w} \) as

\[
\mathbf{v} + \mathbf{w} = \begin{pmatrix} a_1 + a_2 \\ b_1 + b_2 \end{pmatrix}
\]

and given a scalar \( \lambda \), then the scalar product of \( \mathbf{v} \) by \( \lambda \) is

\[
\lambda \mathbf{v} = \langle \lambda x_1, \lambda y_1 \rangle
\]

• We call \( \mathbf{v} \) and \( \mathbf{w} \) parallel if \( \mathbf{v} = \lambda \mathbf{w} \) for some scalar \( \lambda \neq 0 \). Note that parallel vectors point in the same, or opposite direction.

• We can express a line \( L \) through a point \( P = (x_0, y_0) \) with direction vector \( \mathbf{v} = (a, b) \) by

\[
\mathbf{r}(t) = \overrightarrow{OP} + tv = \langle x_0, y_0 \rangle + t\langle a, b \rangle
\]

Where \( O \) is the origin \((0,0)\). This allows us to parametrize the line \( L \) by the parametric equations:

\[
x = x_0 + at, \quad y = y_0 + bt
\]

• The axes in \( \mathbb{R}^3 \) are labeled so they satisfy the right-hand rule: When the fingers of your right hand curl from the positive \( x \)-axis towards the positive \( y \)-axis, your thumb points in the positive \( z \)-direction.
1. Find the components of $\overrightarrow{PQ}$ where $P = (1, -3)$ and $Q = (2, 7)$.
   
   **Answer:** 1 and 10.

2. Let $\mathbf{v} = \langle 6, 9 \rangle$. Which of the following are parallel to $\mathbf{v}$ and which point in the same direction.

   (a) $\langle 12, 18 \rangle$
   (b) $\langle 3, 2 \rangle$
   (c) $\langle 2, 3 \rangle$
   (d) $\langle -6, -9 \rangle$
   (e) $\langle -24, -27 \rangle$
   (f) $\langle -24, -36 \rangle$

   **Answer:** Both, neither, both, parallel, neither, parallel

3. What is the difference between $(a, b)$ and $\langle a, b \rangle$?
   
   **Answer:** The first is a point, the second is a vector.

4. You are likely used to expressing a line as $y = mx + b$. What advantages does this have over the vector representation, what advantages does the vector representation have?
   
   **Answer:** Open ended: The first representation is unique, however it can’t express vertical lines.

5. The concepts for 2-dimensional vectors generalize to 3-dimensional vectors by just adding a third component.

   (a) Express the 3-dimensional vector with components $a, b, c$ in both row notation, and as a sum of standard basis elements $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

   (b) Express the vector $\mathbf{v}$ from point $P = (a_1, b_1, c_1)$ to $Q = (a_2, b_2, c_2)$ as a row vector. What is the distance from $P$ to $Q$? What is the magnitude of $\mathbf{v}$?

   (c) What is the sum of two vectors $\mathbf{v} = \langle a_1, b_1, c_1 \rangle$ and $\mathbf{w} = \langle a_2, b_2, c_2 \rangle$? What is their difference? What is the scalar product of $\mathbf{v}$ by $\lambda$?

   **Answer:** $\langle a, b, c \rangle$ and $\alpha \mathbf{i} + \beta \mathbf{j} + \gamma \mathbf{k}$.

6. Find a vector parametrization for the line the passes through $P = (4, 0, 8)$ with direction vector $\mathbf{v} = (1, 0, 1)$.
   
   **Answer:** $r(t) = 4 + t, 0, 8 + t$. 
7. Find parametric equations for the line perpendicular to the \(xz\)-plane, passing through the point \((1, -1, 2)\).

**Answer:** A vector perpendicular to the plane is \(\mathbf{j}\), so then \(x = 1, y = t - 1, z = 2\) works.

8. Which of the following ways of labeling axes follow the right hand rule?

**Answer:** No, yes, yes, yes.